Coherent lunar effect on solar neutrino:double slitt interference

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Coherent interaction of solar neutrino with the moon and its implication are investigated. We solve the Dirac equation with the moon potential and show that a phase shift of the neutrino wave function becomes almost one unit if the neutrino penetrates the moon. Spatial interference is generated for the neutrino wave packets during eclipse and possibility of observing interference effect that is similar to that of a double slit experiment is pointed out.

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Since neutrino from Supernova were observed [1], neutrino astronomy started. An astronomical observation using neutrino could supply a new information which is not obtained by light. However neutrino interacts with matter so weakly that event rate is extremely small. High luminosity and enhancement of event rate are required to make a real observation possible. The most luminous source of neutrino around the earth is sun and it is practical to use solar neutrino for astronomical purpose. Especially solar neutrino of discrete energy from Be^7 process can be a useful source for astronomical observations. Coherent scattering of neutrino by neutral current interactions can enhance the event rate. We study a coherent moon effect to solar neutrino in this paper.

Coherent interaction of neutrino with massive objects is a unique macroscopic quantum phenenomenon which appears despite an extreme weakness of interaction with matters. The flux of solar neutrino is $10^9/cm^2sec$ for Be^7 neutrino and in a charged current event, which occurs incoherently, a rate is about one hundred per year in Super Kamiokande. A mean free path of 1 MeV neutrino in the earth is about 4×10^{10} Km and is slightly smaller in the moon. The earth and the moon are quite transparent as far as the charged current interaction is concerned. Strength of the neutral current interaction is smaller than the charged current interaction and elastic scattering process $\nu_e + e^- \to e^- + \nu_e$ is similar to the neutral current interaction but has a larger strength. In these processes the neutrino and other matters can stay in almost the same quantum states, and by the coherent sum of amplitudes a total amplitude is enhanced. This effect becomes important when neutrino interacts with a massive object like a star. The moon is a star around the earth and can be located between the earth and the sun. So the coherent interaction of the solar neutrino with the moon can take place. We study the coherent scattering of the solar neutrino with moon in this paper.

The MSW coherent matter effect Ref.[2-3] is actually one of the important ingredients in understanding current neutrino oscillation experiments. Because expected day-night effect has not been observed, the MSW effect would not be important for the earth and for the moon. Double slitt like effect we study in this paper is a spatial interference effect of the lightest neutrino. This effect is sensitive to the absolute magnitude of the lightest neutrino mass.

The solar neutrino has an energy in MeV region and the coresponding de Broglie wave length is $10^{-13}m$,whearas the radius of the moon is of order 10^6 m. Because the wave length is much smaller than the total size, this regime may be thought as a geometrical optics regime or the classical regime for the neutrino where the wave character is normally washed out. However the interaction strength of neutrino with matter is extremely weak and the neutrino propagates almost freely even in matter with keeping an original phase. Consequently the propagation of neutrino in the moon should be treated by quantum mechanical wave equation even in the macroscopic regime. Interference is a key phenomenon of wave dynamics. We show that to generate interference for solar neutrino, wave packet character ,which have been studied for neutrino oscillations in Ref. [4-8], is important. Using reasonable parameter values, we estimate the magnitude of the effect from overlapp between an initial wave packet and a final wave packet. We show that an observation of the effect would be possible if the absolute value of the neutrino mass is $10^{-4}eV$ or less. Observation of the interference should verify quantum mechanics in Gm region and give new informations on the neutrino mass and on the interior of the moon.

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By replacing the quark pair operators or the electron pair operators in the four Fermi interaction with the density of electrons and the quarks, we obtain a potential term in Dirac equation. We solve the equation and find that the effect of the potential arises in the momentum dependent phase factor. Although the effect disappers from the magnitude of a plane wave, the potential gives an interference effect to the wave packet since the wave packet is a linear combination of plane waves. Interference among the plane waves is generated and an observable effect is obtained. It is predicted that the solar neutrino flux changes with time when they are blocked by the moon during eclipses.

potential

Neutrino interacts with quarks $q(x)$ or with the electron $e(x)$ in matter by the charged current interaction

$$\frac{G_F}{\sqrt{2}}\bar{\nu}(x)\gamma_\mu(1-\gamma_5)e(x)\{\bar{d}(x)\gamma^\mu(1-\gamma_5)u(x)+\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu(x)\}, \tag{1}$$

and by the neutral current interaction,

$$\begin{aligned}&\frac{G_F}{\sqrt{2}}\bar{\nu}(x)(1-\gamma_5)\gamma_\mu\nu(x)\{g_V(u)\bar{u}(x)\gamma^\mu u(x)+g_V(d)\bar{d}(x)\gamma^\mu d(x) \\ &+g_V(e)\bar{e}(x)\gamma^\mu e(x)+axialvectors\}.\end{aligned} \tag{2}$$

Throughout this paper we assume that the electron neutrino is lightest and the mixing angle is negligible. The second term in charged current interction has a same final state and initial state as electron neutrino neutral current interaction and by Fiertz transformation the interaction is written as

$$\frac{G_F}{\sqrt{2}}\bar{\nu}(x)(1-\gamma_5)\gamma_\mu\nu(x)\{\bar{e}(x)\gamma^\mu e(x)+axialvector\}. \tag{3}$$

The coefficents $g_V(u), g_V(d), g_V(e)$ are given by the Weinberg angle θ_W as,

$$g_V(u)=\frac{1}{2}-2\times\frac{2}{3}\times\sin^2\theta_W \tag{4}$$

$$g_V(d)=-\frac{1}{2}-2\times(-\frac{1}{3})\times\sin^2\theta_W \tag{5}$$

$$g_V(e)=-\frac{1}{2}+2\times\sin^2\theta_W \tag{6}$$

Using the current value $\sin^2\theta_W=0.23108$ [9], we have

$$g_V(u)=0.19, g_V(d)=-0.35, g_V(e)=-0.034 \tag{7}$$

The charged current interaction,Eq.(1), transforms the neutrino to the electron. Since the final state in the matter is different from the initial state, the process occurs incoherently. The mean free path of the neutrino in the earth from the incoherent scattering $\nu+n\rightarrow e+p$ is computed from the crossection, σ, and the neutron density ρ_n. By using the crosssection and the density,

$$\sigma=\frac{{G_F}^2}{\pi}2M_nE_\nu \tag{8}$$

$$\rho_n=0.556\frac{r}{2}N_{Avo}/cm^3, \tag{9}$$

where r is the specific gravity and N_{Avo} is Avogadoro Number, we have the mean free path,

$$\begin{aligned}l &= \frac{1}{\sigma\rho} \\ &= 10^{11}Km\end{aligned} \tag{10}$$

for $r=5$ and $E_\nu=1$ Mev. Hence the mean free path is much larger than the size of the earth, $10^5 Km$ or the size of the moon, 2×10^3 Km. Neutrino propages in the earth or in the moon almost freely as far as the charged current interactions are concerned.

The strengths of neutral current interactions, Eq.(3), are about the same as the charged current interactions. So a probability of incoherent scattering event from the neutral current interaction is also about the same as the charged current interaction and is very small.

Coherent scattering is possible in the neutral current interactions since the final state in the matter is the same as the initial state. In the coherent scattering, amplitude from each atom is coherently added and a total amplitude can become much larger than the original amplitude. In this case, an enhancement is possible. We will see that the charge of vector current can have a coherent contribution.

To study weak matrix elements, expectation value of the current in the matter

$$\langle Matter|\bar{f}(0)\gamma_\mu \Gamma f(0)|Matter\rangle \tag{11}$$
$$= \sum_i \bar{f}_i \gamma_\mu \Gamma f_i(0)$$

is needed where summation over atoms is taken. Expectation values of the currents in one particle state with a momentum $\vec{p}$ and a spin $\vec{s}$ are

$$\bar{u}(p,s)\gamma_0 u(p,s) = \frac{p_0}{m}, \tag{12}$$
$$\bar{u}(p,s)\gamma_i u(p,s) = \frac{p_i}{m},$$
$$\bar{u}(p,s)\gamma_0\gamma_5 u(p,s) = 0, \tag{13}$$
$$\bar{u}(p,s)\gamma_i\gamma_5 u(p,s) = s_i.$$

Only the zeroth component of the vector charge is positive definite and nearly 1 in the forward scattering region of the neutrinos. Other components are not positive definite and are small. Hence after the summation over whole atoms, only the vector charge becomes non-zero value $\sum_i \bar{u}(p,s)\gamma_0 u(p,s) = \rho$ and others vanish.

Consequently coherent neutrino interaction with matter is obtained by the vector charge and is reduced to an effective two body Hamiltonian,

$$\bar{\nu}(x)\frac{1-\gamma_5}{2}\gamma_\mu \nu(x) V_0(x), \tag{14}$$
$$V_0(x) = \sqrt{2}G_F(\rho_e(x) - 0.5\rho_{neutron}(x)), \tag{15}$$

where $\rho_e(x)$ is the density of the electrons and $\rho_{neutron}(x)$ is the density of the neutron .

plane wave

We solve a Dirac equation with the potential term that is produced by the interaction with matter. Since the potential is caused by the weak interaction, the magnitude is very small and is proportional to the Fermi coupling constant. The range of the potential we study,on the other hand, is very large. So it is interesting to see if the effect of the potential is observable.

Let us solve a Dirac equation with a left-handed potential term,

$$i\hbar\frac{\partial}{\partial t}\psi(x) = (i\vec{\alpha}\cdot\vec{p} + m\beta)\psi(x) + V(x)(\frac{1-\gamma_5}{2})\psi(x), \tag{16}$$

$$V(x) = V_0, r \leq R \tag{17}$$
$$V(x) = 0, r \geq R,$$

here R is a large macroscopic value and V_0 is a small value. We will see that the product $V_0 R$ is order 1.

We obtain a stationary solution of the energy E

$$\psi(x) = \exp{(Et/i\hbar)}\psi(\vec{x}) \tag{18}$$

with a boundary condition at $z \to -\infty$

$$\phi(\vec{x}) = e^{i\vec{k}\cdot\vec{x}}u(\vec{k}), \tag{19}$$

where $u(\vec{k})$ is a free Dirac spinor of a momentum $\vec{k}$. $\psi(\vec{x})$ satisfies an intergral equation,

$$\psi(\vec{x}) = \phi(\vec{x}) + \int d\vec{x}' D(\vec{x}-\vec{x}')V(\vec{x}')(\frac{1-\gamma_5}{2})\psi(\vec{x}'), \tag{20}$$

with the Green function of the Dirac operator,

$$D(\vec{x}-\vec{x}') = \int \frac{d\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} \frac{E+\vec{\alpha}\cdot\vec{p}+m\beta}{E^2-\vec{p}^2-m^2+i\epsilon}. \tag{21}$$

By applying Born approximation,we have an solution

$$\psi(\vec{x}) = \phi(\vec{x}) + \psi(\vec{x})^{(1)} + \psi(\vec{x})^{(2)} + \psi(\vec{x})^{(3)} + -, \tag{22}$$

$$\psi(\vec{x})^{(l+1)} = \int d\vec{x}' D(\vec{x}-\vec{x}') V(\vec{x}') (\frac{1-\gamma_5}{2}) \psi(\vec{x}')^{(l)},$$

$$\begin{aligned}\psi(\vec{x})^{(1)} = {} & e^{i\vec{k}\cdot\vec{x}} \int \frac{d\vec{p}}{(2\pi)^3} e^{i(\vec{p}-\vec{k})\cdot\vec{x}} \frac{1}{\vec{k}^2-\vec{p}^2+i\epsilon} \times \\ & (\frac{1-\gamma_5}{2}(2E(\vec{k})+\vec{\alpha}\cdot(\vec{p}-\vec{k})) + \gamma_5\beta m) u(\vec{k}) \tilde{V}(\vec{k}-\vec{p}),\end{aligned} \tag{23}$$

where

$$\begin{aligned}\tilde{V}(\vec{k}-\vec{p}) &= \int d\vec{x}' e^{i(\vec{k}-\vec{p})\cdot\vec{x}'} V(\vec{x}') \\ &= 4\pi V_0 R^3 v_0(q),\end{aligned} \tag{24}$$

$$\begin{aligned}v_0(q) &= \frac{1}{q}(\frac{\sin q}{q^2} - \frac{\cos q}{q}), \\ q &= R|(\vec{p}-\vec{k})|\end{aligned} \tag{25}$$

Identity,

$$\begin{aligned}&(E(\vec{k})+\vec{\alpha}\cdot\vec{p}+m\beta)\frac{1-\gamma_5}{2}u(\vec{k}) \\ &= \frac{1-\gamma_5}{2} 2E(\vec{k})u(\vec{k}) + \gamma_5 m\beta u(\vec{k}) + \frac{1-\gamma_5}{2}\vec{\alpha}\cdot(\vec{p}-\vec{k})u(\vec{k})\end{aligned} \tag{26}$$

was used in Eq.(24). Finally we have

$$\begin{aligned}\psi(\vec{x})^{(1)} &= e^{i\vec{k}\cdot\vec{x}}\{\frac{1-\gamma_5}{2} 2E(\vec{k}) \frac{V_0 R}{2\pi^2} u(\vec{k}) F(\vec{x},\vec{k}) \\ &+ \gamma_5\beta m \frac{V_0 R}{2\pi^2} u(\vec{k}) F(\vec{x},\vec{k}) + \frac{1-\gamma_5}{2}\frac{V_0}{2\pi^2}\alpha_i F_i(\vec{x},\vec{k}) u(\vec{k})\},\end{aligned} \tag{27}$$

$$F(\vec{x},\vec{k}) = \int d\vec{q} \frac{1}{-2\vec{k}\cdot\vec{q}+i\epsilon} v_0(q) e^{\frac{i\vec{q}\cdot\vec{x}}{R}} \tag{28}$$

$$F_i(\vec{x},\vec{k}) = \int d\vec{q} q_i \frac{1}{-2\vec{k}\cdot\vec{q}+i\epsilon} v_0(q) e^{\frac{i\vec{q}\cdot\vec{x}}{R}}. \tag{29}$$

Eqs. (28) and (29) become

$$F(\vec{x},\vec{k}) = i\frac{4\pi^2}{2k}\sqrt{1-\vec{\xi}_T^2}, \tag{30}$$

$$F_i(\vec{x},\vec{k}) = \frac{\partial}{\partial k_i} F(\vec{x},\vec{k}) \tag{31}$$

in the region $\vec{k}\cdot\vec{x} > R, 1 > \vec{\xi}_T^2$ where $\vec{\xi} = \vec{x}/R, \vec{\xi}_T = \vec{\xi} - \vec{k}(\vec{k}\cdot\vec{\xi})/k^2$. In the region where R and $\frac{E}{m}$ are large, the first term is dominant over the second term and the third term in Eq.(27), and we have

$$\psi(\vec{x})^{(1)} = \frac{1-\gamma_5}{2} 2E(\vec{k}) \frac{V_0 R}{2\pi^2} F(\vec{x},\vec{k}) \phi(\vec{x}). \tag{32}$$

Thus the first order term changes with position in the range R even though the wave length is microscopic size. It should be noted however that this term is pure imaginary. It is suggested that correction terms modify only phase factor of the wave function. We show in the following that this is the case in fact.

The second order term is computed in a similar manner. The dominant term is given as,

$$\psi(\vec{x})^{(2)} = (\frac{1-\gamma_5}{2} 2E(\vec{k})\frac{V_0 R}{2\pi^2})^2 F^{(2)}(\vec{x},\vec{k})\phi(\vec{x}), \tag{33}$$

where the coefficient is given as

$$F^{(2)}(\vec{x},\vec{k}) = \int d\vec{q}_1 d\vec{q}_2 \frac{1}{-2\vec{k}\cdot(\vec{q}_1+\vec{q}_2)+i\epsilon} v_0(q_1) \frac{1}{-2\vec{k}\cdot\vec{q}_2+i\epsilon} v_0(q_2) e^{\frac{i(\vec{q}_1+\vec{q}_2)\cdot\vec{x}}{R}}. \tag{34}$$

By writing the integral with a symmetric manner, we have

$$\begin{aligned} &F^{(2)}(\vec{x},\vec{k}) \\ &= \frac{1}{2!}\int d\vec{q}_1 d\vec{q}_2 \frac{1}{-2\vec{k}\cdot(\vec{q}_1+\vec{q}_2)+i\epsilon}(\frac{1}{-2\vec{k}\cdot\vec{q}_2+i\epsilon}+\frac{1}{-2\vec{k}\cdot\vec{q}_1+i\epsilon}) \\ &\times v_0(q_1)v_0(q_2)e^{\frac{i(\vec{q}_1+\vec{q}_2)\cdot\vec{x}}{R}} \\ &= \frac{1}{2!}\int d\vec{q}_1 d\vec{q}_2 \frac{1}{-2\vec{k}\cdot\vec{q}_1+i\epsilon}\frac{1}{-2\vec{k}\cdot\vec{q}_2+i\epsilon} v_0(q_1)v_0(q_2)e^{\frac{i(\vec{q}_1+\vec{q}_2)\cdot\vec{x}}{R}} \\ &= \frac{1}{2!}F(\vec{x},\vec{k})^2. \end{aligned} \tag{35}$$

Thus the second order term becomes,

$$\psi(\vec{x})^{(2)} = \frac{1}{2!}(\frac{1-\gamma_5}{2} 2E(\vec{k})\frac{V_0 R}{2\pi^2} F(\vec{x},\vec{k}))^2\phi(\vec{x}) \tag{36}$$

and higher order terms are written in the same manner

$$\psi(\vec{x})^{(l)} = \frac{1}{l!}(\frac{1-\gamma_5}{2} 2E(\vec{k})\frac{V_0 R}{2\pi^2} F(\vec{x},\vec{k}))^l\phi(\vec{x}). \tag{37}$$

Adding all higher order terms we have the wave function

$$\psi(\vec{x}) = \exp{(i\vec{k}\cdot\vec{x} + i\frac{1-\gamma_5}{2} 2V_0 \frac{1}{k}\sqrt{\vec{k}^2(R^2-\vec{x}^2)+(\vec{k}\cdot\vec{x})^2})} u(\vec{k}). \tag{38}$$

Substituing the Fermi coupling constant, the radius and the density of the moon assuming the specific gravity r,

$$G_F = 1.16\times 10^{-5}(GeV)^{-2} \tag{39}$$

$$R = 1.74\times 10^3 Km \tag{40}$$

$$\rho_e = \rho_{neutron} = 0.556\frac{r}{2}N_{Avo}/cm^3 \tag{41}$$

to Eq.(15), the numerical constant in Eq.(38) becomes for $r = 5$

$$2V_0 R = 0.95. \tag{42}$$

This is an interesting value to see an interference effect. However the correction is in the phase factor and disappears in the $|\psi(\vec{x})|^2$. So the neutrino flux is the same as free wave if the neutrino is a plane wave. It is impossible to observe the interference effect using any plane wave in the present situation.

wave packet

Realistic solar neutrino is not a plane wave but is a linear combination of plane waves, a finite wave packet Ref.[4-9,11-12]. The scattering amplitude is the overlapp between the initial wave packet and the final wave packet. The former is determined from the beam and the latter is determined from the detector.

From the Eq.(38), the phase of the initial wave function through the moon depends on several parameters such as momentum and position and the wave packet is certain linear combinations of these waves. We will show that the overlapp between these wave packets and the final wave packets have interference effect if the mass of the lightest neutrino is about $10^{-4}eV/c^2$ or less.

A wave packet expands during a propagation [12,13]. Speed of expansion is determined by the velocity variance and is dominant in the transverse direction for a relativistic particle. Using the initial size δx, the maxmimum velocity in

the transverse direction is given as $v_T = \frac{\delta P_T}{E} = \frac{\hbar}{\delta x E}$. The size in the transverse direction is a product of the velocity and the propagation time

$$\Delta x_T = v_T \delta t. \tag{43}$$

The size for the neutrino of $E = 0.6MeV, \delta x = 10^{-10}m$ becomes $3 \times 10^5 Km$ for $\delta t = 500s$ and $10^3 Km$ for $\delta t = 1s$. The former one is the packet size when it propagates from the sun to the earth and the latter is the packet size when it propagates from the earth to the moon. The radius of moon is $1.7 \times 10^3 Km$ and is about the same as the packet size of the neutrino at the moon which is observed at the earth with a microscopic size. So the wave packet effects are relavant for the solar neutrino that is detected at the earth.

The Gaussian wave packet of the variance $\sigma\hbar$ at $t = 0, \vec{x} = \vec{X}_0$ expands at a much later time $t >> t_0$ or at a previous time $t << -t_0$ and behaves at a distant position $\vec{x}$ as

$$\begin{aligned} \psi(\vec{x}, t) &= Ne^{-i\frac{mc}{\hbar}\sqrt{(ct)^2-(\vec{x}-\vec{X}_0)^2}-\frac{1}{2\sigma\hbar}(\vec{P}_X-\vec{p}_0)^2}, \\ \vec{P}_X &= mc\frac{1}{\sqrt{(ct)^2-(\vec{x}-\vec{X}_0)^2}}(\vec{x}-\vec{X}_0), \end{aligned} \tag{44}$$

where N is a normalization factor and $\vec{p}_0$ is the central value of the momentum. From the Gaussian term the wave packet size is determined. The phase factor is written by the use of the momentum as,

$$\phi = \frac{(mc^2)^2}{\hbar|\vec{P}_X c|}t. \tag{45}$$

The phase factor $F(\vec{k}, \vec{x})$ is added for the neutrino which penetrates inside the moon.

We study the overlapp of two wave packets at a certain time at around the moon. Initial wave packet which is emitted at a time T_1 [14] in the sun and propagates toward the earth through the Moon and final wave packet which is detected at a time T_2 at the detector are located around the Moon at a time $t = T_2 - \delta t$ where δt is around one second. The function $F(\vec{k}, \vec{x})$ is in one of the wave function and becomes smooth function of $\vec{x}$ in this region and computation is straightforward.

The initial wave packet which is emitted from the sun at $(\vec{X}_1, T_1)$ behaves at $(\vec{x}, t)$ as,

$$\begin{aligned} \psi_{in}(\vec{x}, t) &= Ne^{-i\frac{mc}{\hbar}\sqrt{(c(t-T_1))^2-(\vec{x}-\vec{X}_1)^2}-\frac{1}{2\sigma\hbar}(\vec{P}_{X_1}-\vec{p}_0^{\,in})^2}, \\ \vec{P}_{X_1} &= mc\frac{1}{\sqrt{(c(t-T_1))^2-(\vec{x}-\vec{X}_1)^2}}(\vec{x}-\vec{X}_1). \end{aligned} \tag{46}$$

The final wave packet detected at $(\vec{X}_2, T_2)$ behaves at the same $(\vec{x}, t)$ as,

$$\begin{aligned} \psi_{out}(\vec{x}, t) &= Ne^{-i\frac{mc}{\hbar}\sqrt{(c(T_2-t))^2-(\vec{x}-\vec{X}_2)^2}-\frac{1}{2\sigma\hbar}(\vec{P}_{X_2}-\vec{p}_0^{\,out})^2+F(\vec{x})}, \\ \vec{P}_{X_2} &= mc\frac{1}{\sqrt{(c(T_2-t))^2-(\vec{x}-\vec{X}_2)^2}}(\vec{x}-\vec{X}_2)., \end{aligned} \tag{47}$$

We have chosen the situation where the wave front of the initial wave packet does not reach the Moon but the wave front of the final wave packet reaches the Moon. The phase in Eqs.(46) and (47) depend on the absolute value of the neutrino mass and momentum. For a neutrino of the mass $10^{-4} eV/c^2$ and the momentum $1MeV/c$, the phase factor ϕ is written as

$$\begin{aligned} \phi &= 5\frac{|\vec{x}|}{x_0} \times 10^{-3}, \\ x_0 &= 1000Km, \end{aligned} \tag{48}$$

and becomes negligbly small in the scale of Moon. So the phase factor of $\psi_{in}(\vec{x})$ is constant ϕ_0 and the phase factor of $\psi_{out}(\vec{x})$ is negligibly small in the same scale. Gaussian factors are almost one in the same region. Then overlapp of the two wave functions is given as,

$$(\psi_{out}(\vec{x}, t), \psi_{in}(\vec{x}, t)) = e^{i\phi_0} \int d\vec{r} N e^{iF(\vec{r})-\frac{1}{2\sigma\hbar}(\vec{P}_{X_1}-\vec{p}_0^{\,in})^2-\frac{1}{2\sigma\hbar}(\vec{P}_{X_2}-\vec{p}_0^{\,out})^2}. \tag{49}$$

Since the Gaussian terms are positive definite the integral is reduced by the effect of phase factor $F(\vec{x})$. The reduction rate depends on parameters such as wave packet sizes, the energy and others. Using the Moon radius for the size of detecting wave packet around Moon we have the neutrino flux at about 0.85 of the normal flux value for $\vec{p}_0^{\,in} = \vec{p}_0^{\,out} = \vec{P}_X$ at a total solar eclipse. Reduction is almost the same for other momenta.

We studied coherent scattering of solar neutrino by the moon and found that the moon modifies the phase of solar neutrino wave function. The result is surprizing from two reasons. First one is extreme weakness of potential and second one is the fact that the wave character is seen in extremely large scale. They became possible because, despite extreme weakness of potential, the volume of finite potential region is very large and the product of two values is of order one. The other feature is seen from the fact that the ratio between the distance and de Broglie wave length is around 10^{20}. Normally in this regime the wave character are washed away due to a rapid oscillation of phase and geometrical optics regime is realized. Interference should have not been produced in this case. However in the present case, interference is in fact produced. This is because the time dependent phase and space dependent phase of the relativistic waves cancell and consequently the phase difference in large distance is not washed away and wave phenomena occurs.

As an implication, we suggests that the time dependent moduration of solar neutrino flux occur during eclipse if the suitable detector is used. If a macroscopic wave phenomenon of neutrino is verified, this can be used for testing quantum mechanics in the scale of Gmeter range. New informations on the absolute value of the lightest neutrino mass and the interior of the moon could be supplied also.

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